catch the light

technical reprint R/P063

basic physics & statistics of photomultipliers

electron tubes

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1 introduction

This paper explains how the basic physical processes occurring in photomultipliers determine their performance and refers to what might be called a theoretician's photomultiplier in that no account is taken of the practical engineering problems encountered in making any device. The theory thus refers to ideal performance whilst a real photomultiplier will necessarily behave in a more complicated and possibly less satisfactory manner.

If we consider a photomultiplier as part of a system we can describe it as a transducer which converts an optical signal into an electrical one and then amplifies it. The only parameters required will then be the photocathode conversion efficiency and the current gain of the multiplier system. Unfortunately this approach rarely provides the user with a sufficient understanding, because the two basic processes involved, photo-emission and secondary emission, are essentially quantum ones. It is convenient to consider each of these briefly before relating the quantum ideas of photons and electrons to those of light flux and current.

2 photo-emission

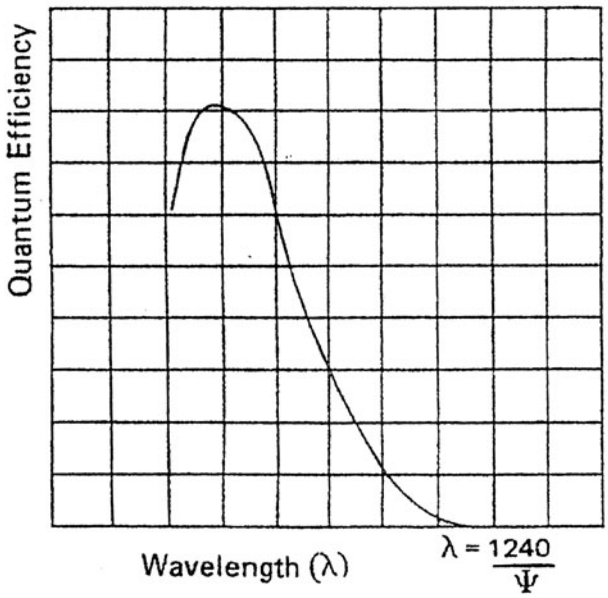
Einstein's law states that the kinetic energy of E of the electron emitted when a photon of frequency is ν is incident on a surface with work function is given by:

$$E = h\nu - \Psi \text{ (h is Planck's constant)}$$

Writing this in units of eV and measuring the wavelength of the photon in nm gives:

$$E = \frac{1240}{\lambda} - \Psi$$

This equation indicates the greatest possible wavelength at which emission might occur but says nothing about the efficiency η for producing electrons at shorter wavelengths. In fact η only rises very slowly and, for most cathode materials, has the general form:



At shorter wavelengths the response is usually limited by the transparency of the tube envelope rather than by the cathode sensitivity.
There is no detailed theory which can be used to predict this curve but its general features can be understood in terms of the need for the photon to be absorbed very close to the surface and for the electron to have both the right component of velocity and sufficient excess energy to escape. The thickness of semi-transparent cathodes is clearly critical and in practice is about 20 nm. It is hardly surprising that the maximum quantum efficiency is less than 0.5 and the best current values, ~0.3, are not expected to improve very much.

If the light is traveling at an angle its path length in the cathode is increased and therefore η can be rather larger than for normal incidence. There is also a polarization effect as light with its electric field component directed out of the plane of the surface is more effective in ejecting electrons. These effects are not well documented but are probably of the order of 10%.

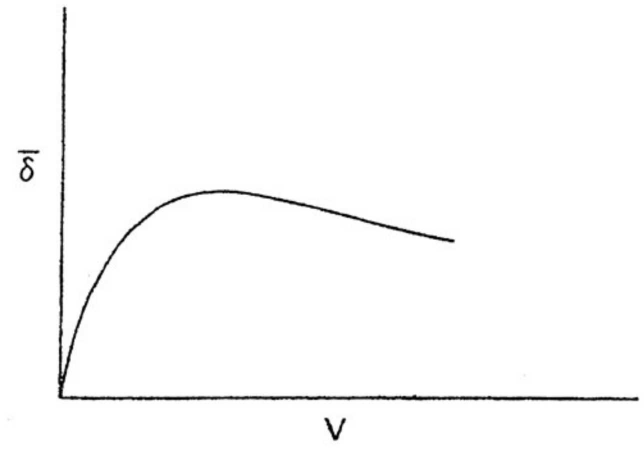
The electrons are emitted in all directions and have a distribution of energy with a cut-off at $(h\nu - \Psi)$. Note that this average energy will be greater for incident light of shorter wavelength. The emission from available cathode materials always occurs very rapidly, $<10^{-11}$s.

The photons arriving at the cathode behave independently of each other and all have the same probability of releasing an electron. (This is not true for very intense beams from pulsed lasers but as the cathode would be destroyed by the beam this case need not be considered). The response is therefore linear, provided that the current which flows does not alter the potential distribution. It is also follows that, if N/η photons are incident, it is only the mean number of electrons produced with is equal to N. The problem is similar to the basic one in probability theory which applies to tossing dice or drawing cards from a pack - the statistics of the number of photo-electrons must be given by the binomial distribution. This will apply exactly, except possibly in the ultra violet region of the spectrum where it is energetically possible for an individual photon to produce more than one electron.

3 secondary emission

The variation of the mean number of secondary electrons $\bar{\delta}$ with the energy of the incident electron has the same general shape for all surfaces, although numerical values differ widely:

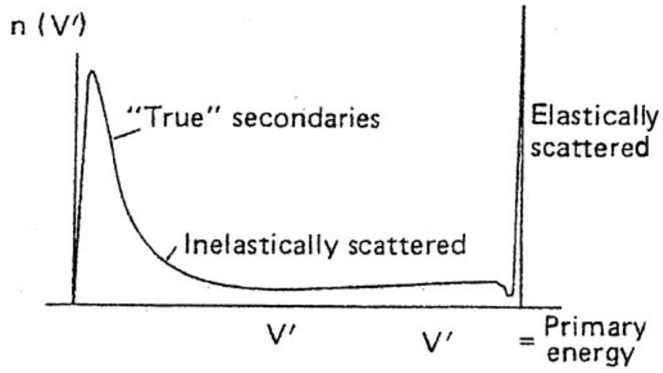
The rising part of the curve reflects the increase in the energy available but as the primary electron buries itself more deeply in the dynode, the secondary electrons are less likely to escape and thus the curve turns over. The same argument suggests that oblique incidence would lead to a higher maximum and this is indeed so. In most practical tubes the steeper part of the curve is used where δ is increasing at a rate between $V^{0.6}$ and $V^{0.8}$.



The secondary electrons emerge in all directions and have a wide distribution of energies:

About 1% of the incident electrons may be elastically scattered at the dynode and appear as very high energy secondaries while a rather larger proportion lose some of their energy by inelastic scattering. The true secondries have a most probably energy in the region of 2eV. Unfortunately the exact form of this curve is not well established for the dynode materials used in most commercial photomultipliers. It is again true that the emission can be considered instantaneous.

If each primary electron arrives with the same energy, it contributes, on average, the same number of secondary electrons. Each electron acts independently, so it follows that the secondary current is exactly proportional to the primary current.



Although the average number of secondary electrons can readily be measured, the statistical distribution is a much more intractable problem because it can only be determined indirectly. Theoretical discussion is not very helpful; if each secondary electron required a particular energy to be emitted then $\bar{\delta}$ might be the same for every event but if the depth of penetration of the primary electron depended critically on whether it made a large angle scatter near the surface then the distribution of $\bar{\delta}$ might be very broad. It is customary to assume that the distribution is a Poisson one but it must be stressed that this assumption has no theoretical basis.

4 photomultiplier response

For an idealised photomultiplier we can assume that η and $\bar{\delta}$ are constant on the surface of each electrode and also that all the electrons emitted are collected by the following stage. If the voltage between each pair of dynodes is the same the output charge for each electron leaving the cathode is just $\delta^n.e$ for a n stage tube. Writing $G=(\delta)^n$ the output charge for N photoelectrons is then $q = Gne$ (problems due to space charge which, particularly if N is large, may arise in the latter stages of the tube, are ignored in this treatment, as are effects due to the finite transit times of the electrons). If the quantum efficiency of the cathode, as a function of wavelength, $\eta(\lambda)$, is known then the response to any number of incident photons is completely established.

For use in measuring a light flux the quantum efficiency of the cathode, $\eta(\lambda)$, needs to be transformed into the radiant power efficiency, $\eta'(\lambda)$.

As photon energy $= \dfrac{1.987\times10^{-25}}{\lambda}$ joules(λ in m) a flux of 1 photon s^{-1} = $\frac{1.987x10^{-16}}{\lambda}$ *watts (λ in nm)*

and 1 electron s^{-1} = *1.602×10^{-19} A,*

we get $\eta'(\lambda) = 0.806\ \lambda\ \eta(\lambda) mAW^{-1} (\lambda \text{ in nm})$

Although it is natural for physicists and enquirers to measure radiant energy in watts, the appropriate unit for optics is lumen. This is essentially a physiological unit because it provides a measure of the optical sensation produced; its relationship to the watt depends on the spectral properties of the radiation considered. Hence the lumen is defined in terms of the relative sensitivity of the eye V(λ) and the luminous efficiency of a cathode is only meaningful for a given source (usually black-body radiation at 2856k).

If the source distribution is $I(\lambda)\ Wm^{-1}$

flux in lumens = $608 \int I(\lambda)\ V(\lambda)\ \delta\lambda$

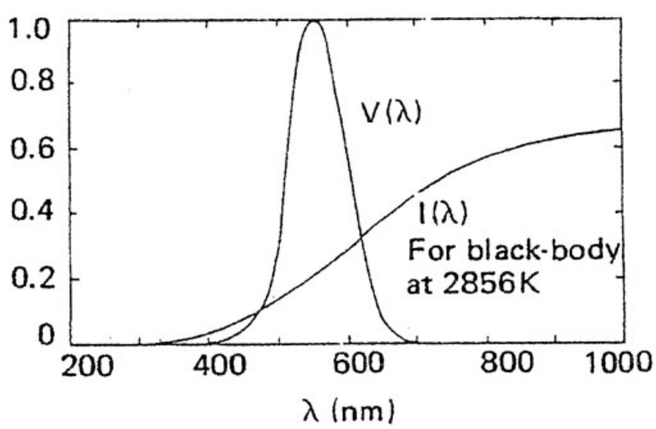
photo-current = $\int I(\lambda) \eta'(\lambda)\ d\lambda$

hence luminious efficiency =

$$1.18 \frac{\int I(\lambda)\ \lambda\ \eta(\lambda)\ d\lambda}{\int I(\lambda)\ V(\lambda)\ d\lambda}\ \mu A lm^{-1}$$

The curves for $V(\lambda)$ and $I(\lambda)$ are given below: only relative values of $I(\lambda)$ are important bcause it appears in both integrals.
Curves for $\eta(\lambda)$ or $\eta'(\lambda)$ must be taken from data sheets or obtained by calibrating individual tubes.

It is important to note that the above calculation cannot be done in reverse. Even the average value of η cannot be deduced from a knowledge of the luminous efficiency. Indeed the latter can be a very poor guide to the maximum quantum efficiency because the form of $I(\lambda)$ makes the luminous efficiency extremely sensitive to the magnitude of the long wavelength tail of $\eta(\lambda)$.



5 fluctuations in response

We must distinguish between fluctuations in 1) the number of output pulses per second and 2) the size of output pulses.

6 fluctuations in number

The quantum nature of light implies that a "steady" flux of light, of mean $\varphi\ s^{-1}$, will exhibit fluctuations about this mean. If these fluctuations are governed by Poisson statistics then the number of photo-electrons, $N = \eta\varphi\ s^{-1}$, will also be distributed in the same way. This case applies in most applications of photomultipliers but even if φ has no fluctuations the standard deviation of N, determined by the binomial statistics of photo-emission is

$$\sqrt{\phi\eta(1-\eta} = \sqrt{N(1-\eta)}.$$

Since η is always less than 0.3 the error in assuming that the deviation = $\sqrt{N}$ is not very large.

The output current is then
$I = Ge\ (N \pm \sqrt{N}\ )$ if measured for 1 second.

Alternatively this problem can be treated using the "shot" noise formula:

$$\overline{i^2} = 2qI\Delta f$$

The charge per pulse q = Ge and I = GeN

$$\therefore \sqrt{i_2} = Ge\ \sqrt{2\Delta f}\ \sqrt{N}$$

This gives the same result as before for a bandwidth = 0.5Hz. Measuring or integration time is related to bandwidth by a Fourier transform so the equivalence of these two results is to be expected (the above analysis could of course be used to derive the "shot" noise formula).

7 fluctuations in amplitude

The mathematical problem is to deduce the distribution of pulse "heights" at the anode for a given number of photons incident simultaneously (i.e., within the resolving time of the recording equipment). Although an analytical solution is possible, it is more instructive to follow the development of the electron cascade by calculating the probability of finding any number of electrons at each stage. The diagrams show the results for the case of a photomultiplier with $\eta = 0.2$, $\delta = 4$ (no variation) and 1, 5 and 20 incident photons. This is very much an idealized photomultiplier so the actual values are not of much significance but the following general points arise from examining the diagrams overleaf.

i) If the number of photons per event is small some events are "lost " both at the cathode and, to a decreasing extent, at the first few dynodes.
ii) Most of the broadening in the distributions take place at the cathode or first dynode and the output pulse height distribution is almost indistinguishable from that after the first stage. It is a good approximation to assume that the relative standard deviation of the out put distribution is equal to

$$\frac{1}{\sqrt{N}} \sqrt{\frac{\delta_1}{\delta_1 - 1}}$$

where δ_1 is the gain at the first dynode.

iii) Observations of the output pulse can never be expected to reveal the number of incident pho tons if this number is small.

iv) The output distribution for single incident photons (and for thermionic noise) is such that the output rate will becritically dependent on the sensitivity of the discriminator.

v) The maxima of the distributions occur at values rather less than the formula for mean gain suggests.

vi) The fluctuations in amplitude lead to an increase in the "shot" noise power by a factor

$$\frac{\delta_1}{\delta_1 - 1}$$

It must be stressed again that these result are for an ideal photomultiplier. Results for real tubes ae often substantially different for small signals; in particular, the single photon output distribution often fails to show a maximum but is monotonically decreasing.

references

The following articles provide very much deeper treatments of the main topics in this paper:

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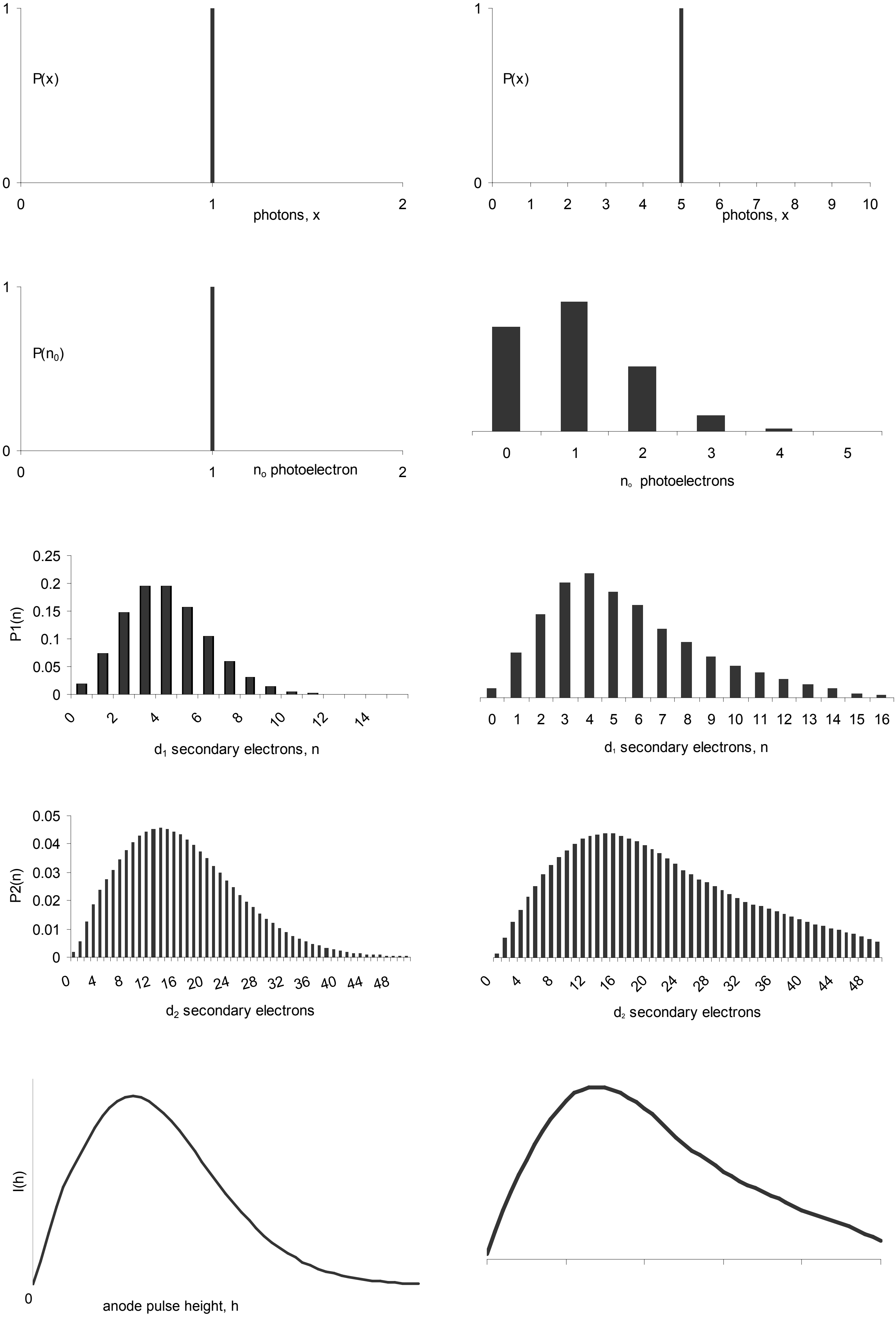
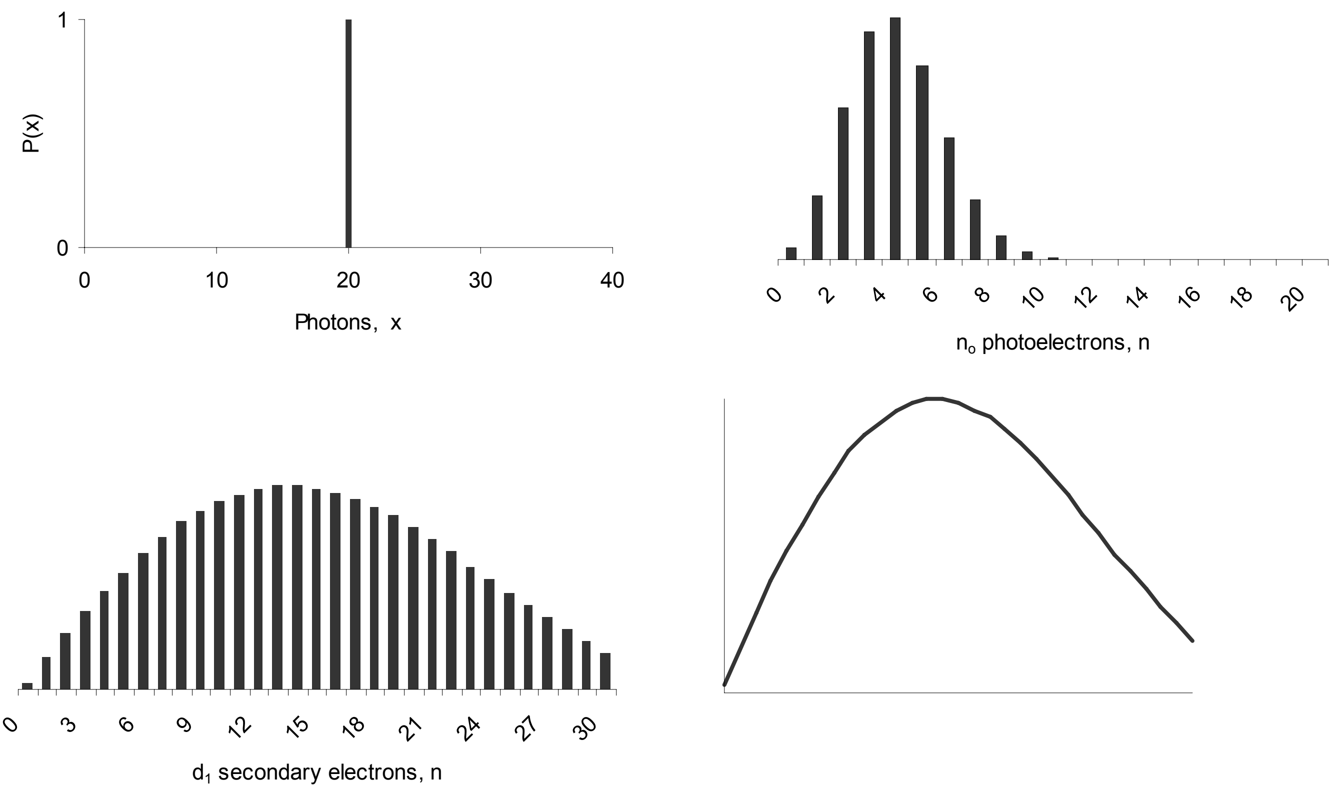

P(x)
photons, x
P(n_0)
n_o photoelectron
n_o photoelectrons
P1(n)
d_1 secondary electrons, n
P2(n)
d_2 secondary electrons
I(h)
anode pulse height, h



data calculated for η = *0.2*, δ = *4*

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